How to Synchronize Efficiently

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Homomorphic Secret Sharing

- Introduced by Boyle, Gilboa and Ishai [BGI] (CRYPTO'16) as a (practical) alternative to FHE
- HSS allows homomorphic evaluation of a function to be distributed among **two parties** who **do not interact** with each other
- BGI constructed a **group based** HSS scheme
  - For functions f described by a branching program

Homomorphic Secret Sharing –cont.

- Received **Best Paper Award** at CRYPTO'16
- Follow-up works: Eurocrypt'17, ACM-CCS'17, ProvSec 17, ITCS'18
- Applications:
  - Private information retrieval (PIR) construction
  - Secure MPC with minimal interaction
  - Secure data access
  - Correlated randomness generation

A main open problem in HSS

- Scheme based on **share conversion** procedure which may **err**
- **Mathematical formulation of main problem (in generic group model)**:

We are given n random numbers arranged in a line. Two parties start in two **adjacent** places, but don't know which one is the first. Each party can query at most T numbers.

The goal of the players is to **synchronize**: choose the same number without any communication.

- **Question**: What is the minimal error probability (as a function of T)?

[BGI16] Solution

15 77 11 104 68 39 94 53 33

- Each party queries T consecutive points and chooses minimum
- Assume T=5

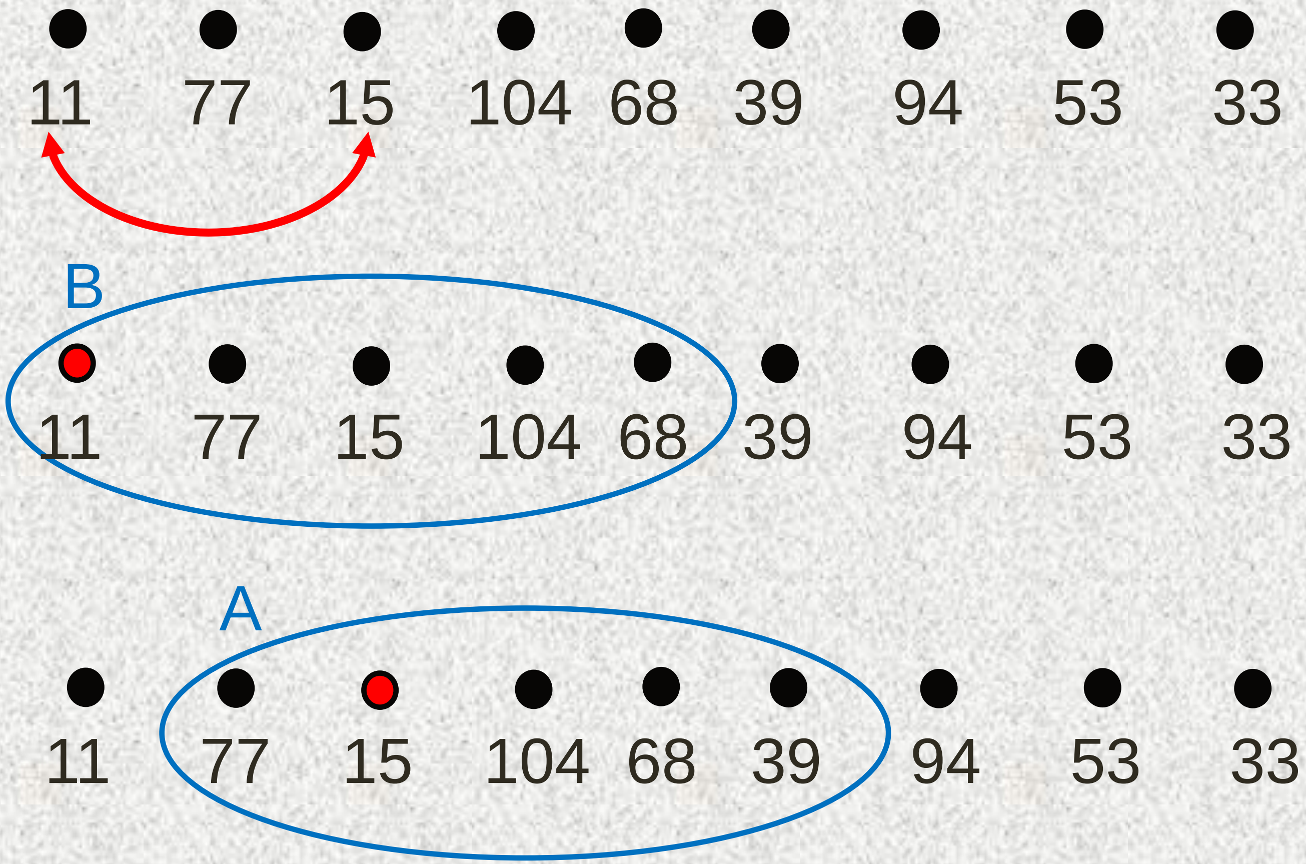
A

15 77 11 104 68 39 94 53 33

B

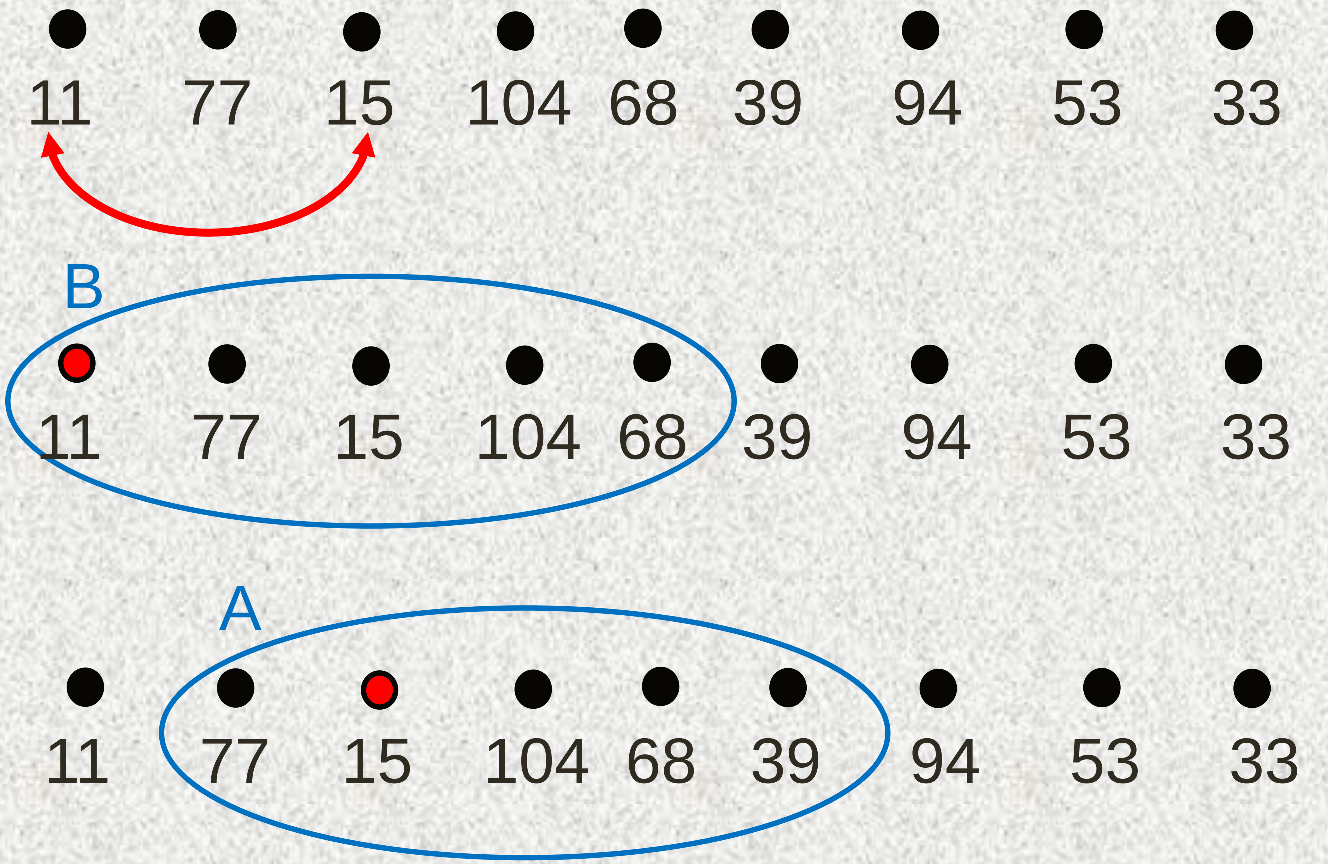
15 77 11 104 68 39 94 53 33

[BGI16] Solution



- Error occurs if minimum is **on the edge**
- Error probability about 1/T

[BGI16] Solution



- **[BGI16]** Error rate of O(1/T)
- **Subsequent papers:** No asymptotic improvement

Our results

- **An algorithm** which achieves $O(1/T^2)$ error rate

- **A matching lower bound (in cryptographic groups):** Result is optimal, unless DLOG in a **short** interval *I* can be solved faster than in $O(\sqrt{|I|})$ operations.
  - Currently not possible for standard cryptographic groups

- **Our techniques:**
  - Random walks (complex variants of Pollard's Kangaroo method)
  - Martingales (algorithm analysis)
  - Discrete Fourier Analysis (lower bounds)

Applications

- **Asymptotic improvement** of computational complexity of the BGI HSS scheme
  - Relevant to applications such as PIR
- Non-cryptographic applications (work in progress with Boyle, Gilboa and Ishai)
  - String algorithms
  - Boolean functions

- **Full paper:** to appear at **CRYPTO'18.**

Thanks for listening!